7-3 Day 4 Volume : Cross Sections

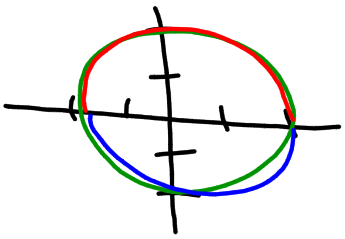
Learning Targets

I find the volume of a solid that has been using the cross sections on base method.

Jan 7-8:47 AM

Ex1. The base of a solid is the circle $x^2 + y^2 = 4$. Find the volume of

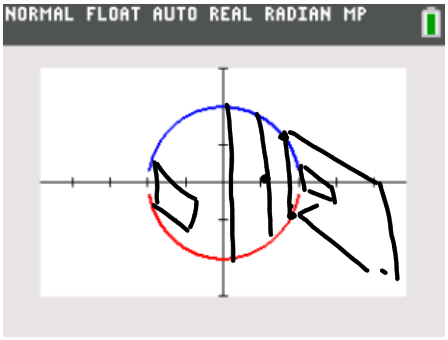
a.) The solid with square cross sections.

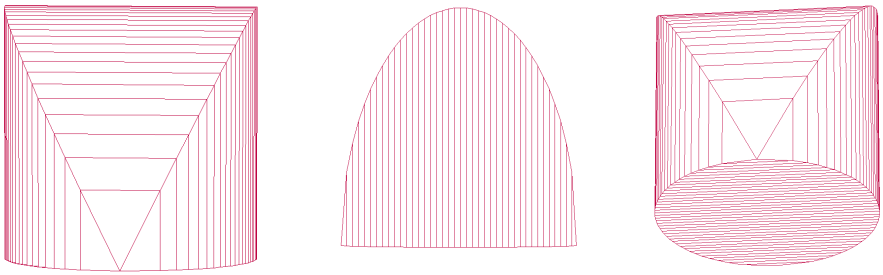


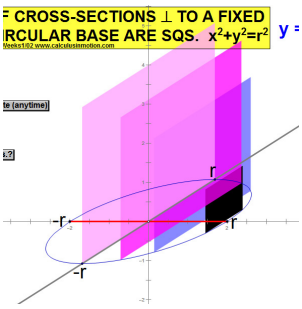
$x^2 + y^2 = 4$

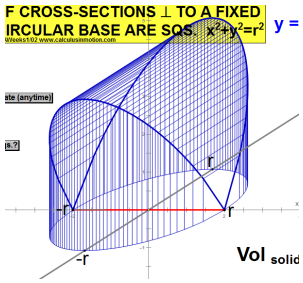
$y^2 = 4 - x^2$

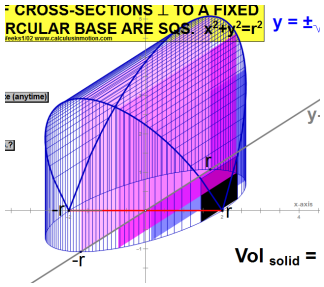
$y = \pm\sqrt{4 - x^2}$

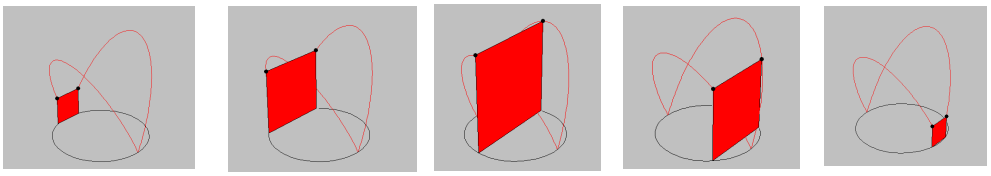


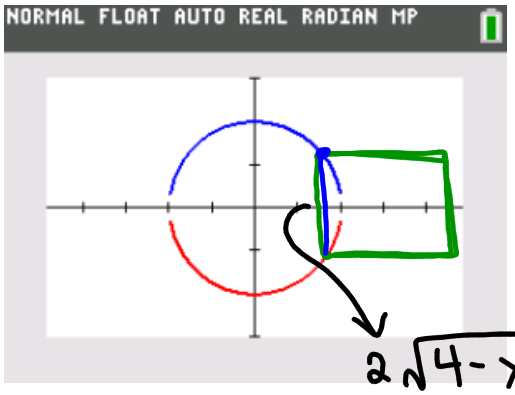












$A = b^2$

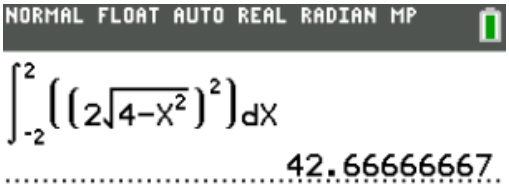
$A = \left(2\sqrt{4-x^2}\right)^2$

$A = 4\left(4-x^2\right)$

$A = 16 - 4x^2$

$\frac{\sqrt{4-x^2}}{2\sqrt{4-x^2}} + \frac{1}{\sqrt{4-x^2}}$

$V = \int_{-2}^{2} \left(2\sqrt{4-x^2}\right)^2 dx$



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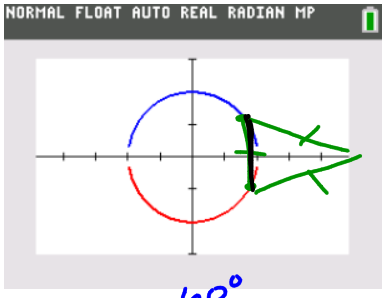
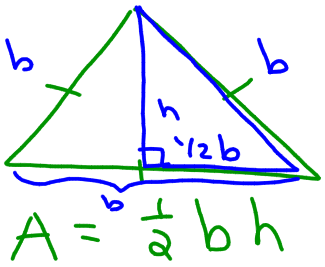
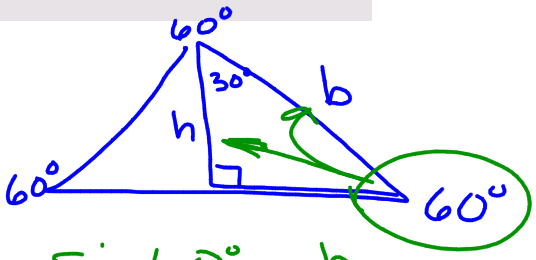
b.) The solid with equilateral triangular cross sections





$A = \frac{1}{2} b h$



$\sin 60° = \frac{h}{b}$

$\frac{\sqrt{3}}{2} = \frac{h}{b}$

$\frac{\sqrt{3}}{2} b = h$

$\left(\frac{1}{2}b\right)^2 + h^2 = b^2$

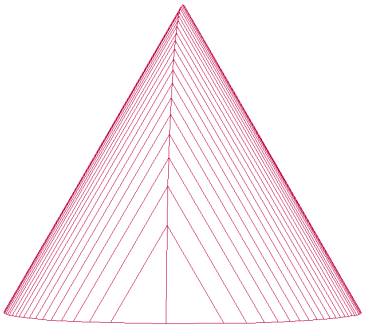
$\frac{1}{4}b^2 + h^2 = 1b^2$

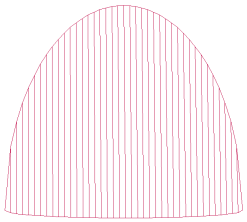
$-\frac{1}{4}b^2 \quad -\frac{1}{4}b^2$

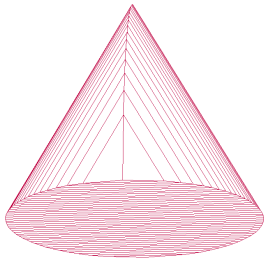
$h^2 = \frac{3}{4}b^2$

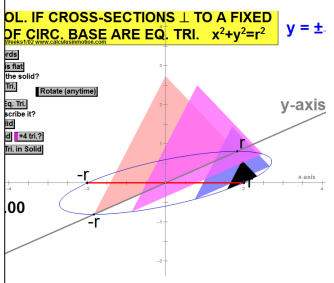
$h = \sqrt{\frac{3}{4}b^2}$

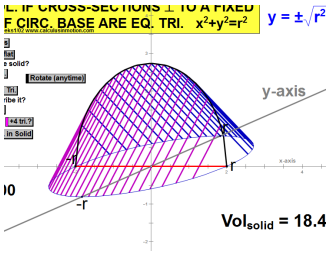
$h = \frac{\sqrt{3}}{2} b$

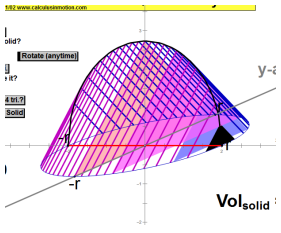


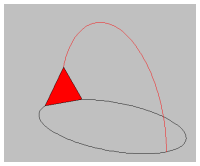


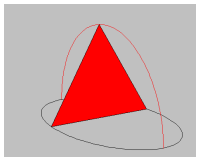


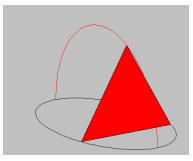


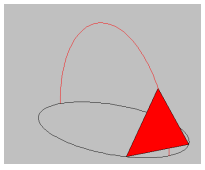












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$A = \frac{1}{2} b \cdot h$

$A = \frac{1}{2} b \cdot \frac{\sqrt{3}}{2} b$

$\boxed{A = \frac{\sqrt{3}}{4} b^2}$

$A = \frac{\sqrt{3}}{4} b^2$

$A = \frac{\sqrt{3}}{4} \left(2\sqrt{4 - x^2}\right)^2$

$\int_{-2}^{2} \frac{\sqrt{3}}{4} \left(2\sqrt{4 - x^2}\right)^2 dx$

by hand: N/C

$\frac{\sqrt{3}}{4} \int_{-2}^{2} 4\left(4 - x^2\right) dx$

$= \sqrt{3} \int_{-2}^{2} 4 - x^2 \, dx$

$= \sqrt{3} \left[4x - \frac{1}{3}x^3\right]_{-2}^{2}$

$= \sqrt{3} \left[\left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right)\right]$

$= \sqrt{3} \left[8 - \frac{8}{3} + 8 - \frac{8}{3}\right]$

$= \sqrt{3} \left[16 - \frac{16}{3}\right]$

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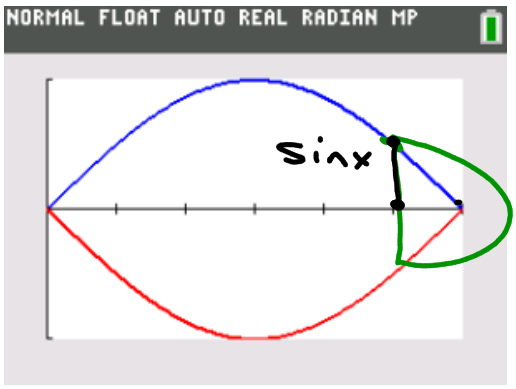
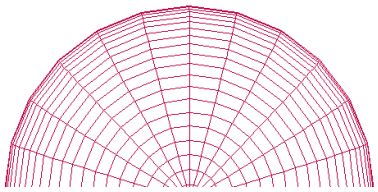
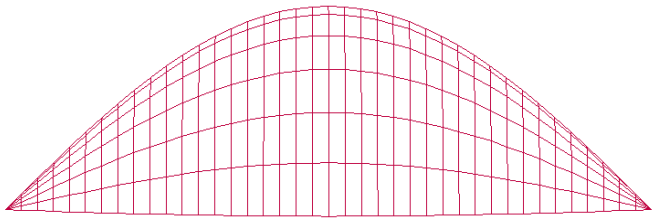
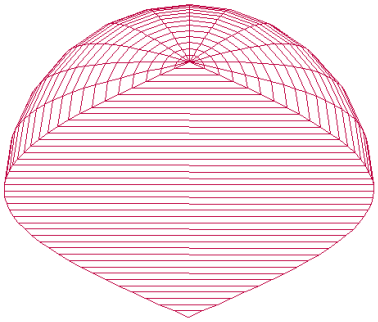
Ex 2 $y = \sin x$
$y = -\sin x$









semi-circular cross sections

$A = \frac{1}{2}\pi r^2$

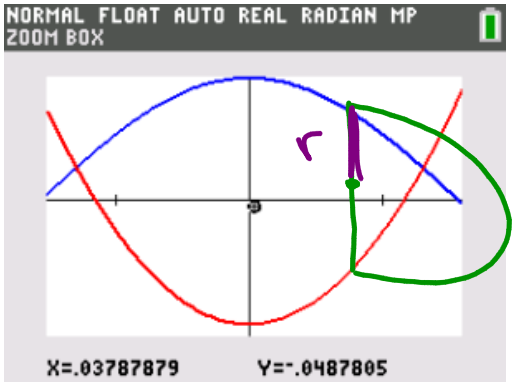
$A = \frac{1}{2}\pi (\sin x)^2$

$V = \int_0^{\pi} \frac{1}{2}\pi \sin^2 x \, dx$

≈ 2.467

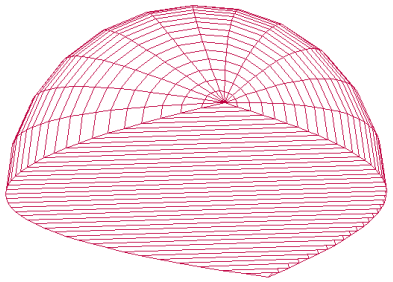
Ex2. The base of a solid is the area bounded by the curves y=cos(x) and $y = \frac{3}{4}x^2 - 1$. Find the volume of the solid with semicircular cross sections.

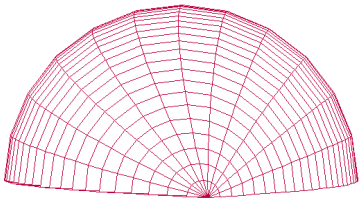
$A = \frac{1}{2}\pi r^2$

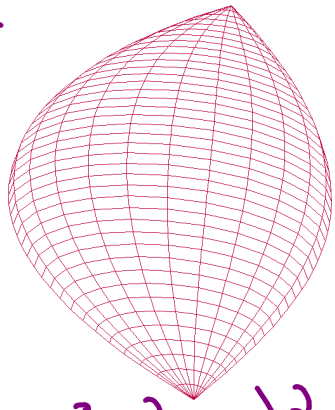


$r = \frac{\cos x - \left(\frac{3}{4}x^2 - 1\right)}{2}$

$r = \frac{\cos x - \frac{3}{4}x^2 + 1}{2}$







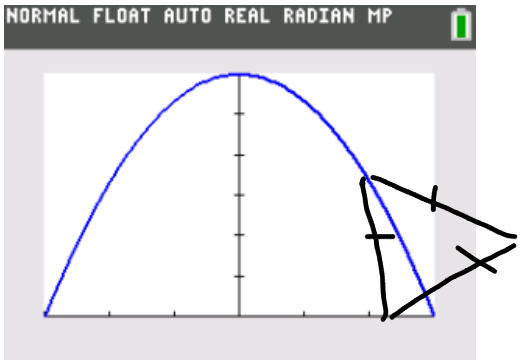
$V = \int_{-1.29996}^{1.29996} \frac{1}{2}\pi \left(\frac{\cos x - \frac{3}{4}x^2 + 1}{2}\right)^2 dx$

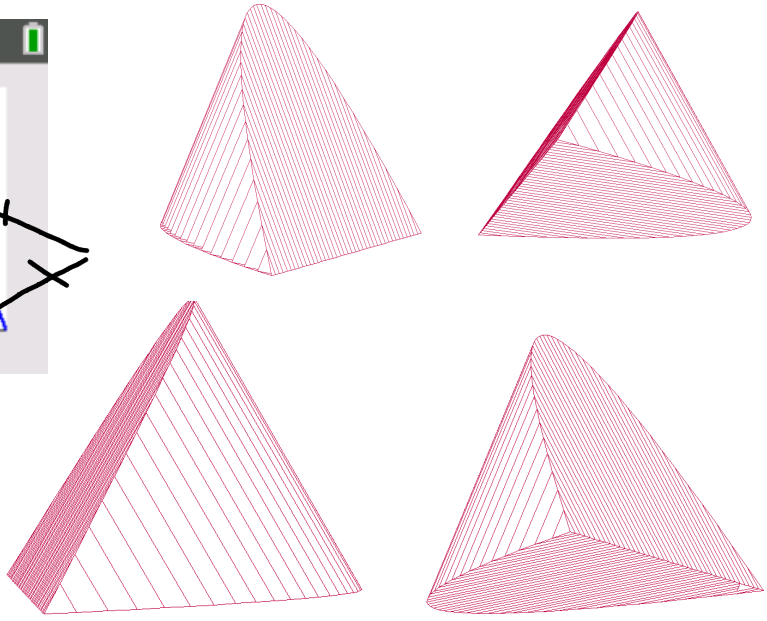
$V = 2.144$

Ex4. The base of a solid is the area bounded by the curve $f(x)=6-\frac{2}{3}x^2$ and the x-axis. Find the volume of the solid with:

a.) Equilateral Triangular Cross Sections

b.) Isosceles Right Triangular Cross Sections (with the hypotenuse on the base).





a.) $A=\frac{\sqrt{3}}{4}b^2$ $b=6-\frac{2}{3}x^2$

$A=\frac{\sqrt{3}}{4}\left(6-\frac{2}{3}x^2\right)^2$

$\int_{-3}^{3}\frac{\sqrt{3}}{4}\left(6-\frac{2}{3}x^2\right)^2 dx \approx 12.068$

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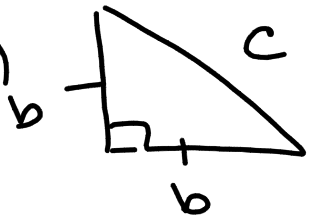
b.)



$A = \frac{1}{2} b \cdot h$

$= \frac{1}{2} \cdot b \cdot b = \frac{1}{2} b^2$

$a^2 + b^2 = c^2$

$b^2 + b^2 = c^2$

$2b^2 = c^2$

$b^2 = \frac{c^2}{2}$

$b = \sqrt{\frac{c^2}{2}} = \frac{c}{\sqrt{2}}$

$A = \frac{1}{2} \cdot b \cdot h$

$= \frac{1}{2} \cdot \frac{c}{\sqrt{2}} \cdot \frac{c}{\sqrt{2}}$

$A = \frac{1}{2} \cdot \frac{c^2}{2} = \frac{1}{4} c^2$

$c = 6 - \frac{2}{3} x^2$

$A = \frac{1}{4} \left(6 - \frac{2}{3} x^2\right)^2$

$$\int_{-3}^{3} \frac{1}{4} \left(6 - \frac{2}{3} x^2\right)^2 dx = 28.8$$

Homework

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